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CORRECTION

Correction to Tuerlinckx, Maris, Ratcliff, and De Boeck (2001)

Francis Tuerlinckx[1]

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Abstract
This erratum identifies and corrects an error in the proof of Appendix A of Tuerlinckx et al. (*Behavior Research Methods, Instruments, & Computers*, *33*(4), 443–456, 2001).

In our paper (Tuerlinckx et al., 2001) on the comparison of four methods for simulating the diffusion process, the proof in Appendix A (of the crucial inequality of the rejection algorithm that the first-passage time density is dominated by a scaled exponential density) is false. The argument in the proof is invalid because the absolute values of subsequent terms in the infinite sum are not monotically decreasing.

In Tuerlinckx et al. (2001), a rejection algorithm is applied to sample from the first-passage time of a Wiener process (with non-zero drift) and with a starting point halfway between two absorbing boundaries. The candidate generating density that is used to dominate the first-passage time density is an exponential density. Therefore, we have to show that this first-passage density can be dominated for all $t > 0$ with a scaled exponential density.

For a formal proof, the setup and notation used by Drugowitsch (2016) will be adopted here. Assume a Wiener process with drift μ, unit variance, two absorbing boundaries located at -1 and 1 and the starting point halfway (i.e., at 0). This can be done without loss of generality.

If we take $f(t)$ to be the density of the first-passage time at the lower (or upper) boundary, then Drugowitsch (2016) shows that (see Equations 15, 18, and 19):

$$f(t) \leq \frac{\pi(e^{\mu}+e^{-\mu})}{4} e^{-\frac{(4\mu^2+\pi^2)t}{8}} =: s_1(t) \quad \text{for } t > \frac{\log\frac{5}{3}}{2\pi^2} \approx 0.026.$$

In the notation of Tuerlinckx et al. (2001), the setup in this erratum means that the boundaries are located at 0 and $a = 2$, the starting point at 1 and variance $\sigma^2 = 1$ (this implies a simple shift of the process). Therefore, the rate of the exponential candidate generating density, denoted λ in Tuerlinckx et al. (2001) (see Equation 13), becomes:

$$\lambda = \frac{\mu^2}{2\sigma^2} + \frac{\pi^2\sigma^2}{2a^2} = \frac{\mu^2}{2} + \frac{\pi^2}{8}.$$

The constant M with which the candidate exponential density should be multiplied in order to dominate the first-passage density is (see Equation 12 in Tuerlinckx et al., 2001):

$$M = \frac{\frac{\pi\sigma^2}{a^2}\left(e^{\frac{a\mu}{2\sigma^2}} + e^{-\frac{a\mu}{2\sigma^2}}\right)}{\frac{\mu^2}{2\sigma^2} + \frac{\pi^2\sigma^2}{2a^2}} = \frac{\frac{\pi}{4}\pi(e^{\mu}+e^{-\mu})}{\frac{\mu^2}{2} + \frac{\pi^2}{8}}.$$

As a consequence, $s_1(t)$ is a scaled exponential density, because $s_1(t) = M\lambda e^{-\lambda t}$.

If we now can show that $f(t) \leq s_1(t)$ for $t \leq \frac{\log\frac{5}{3}}{2\pi^2} := c$ as well, then we have proven the crucial inequality. To proceed, we first take another inequality by Drugowitsch (2016):

$$f(t) \leq \frac{e^{\mu}+e^{-\mu}}{\sqrt{2\pi}} t^{-\frac{3}{2}} e^{-\frac{1}{2t}-\frac{\mu^2 t}{2}} =: s_2(t) \quad \text{for } t < \frac{8}{\log\frac{5}{3}} \approx 15.661.$$

If we can demonstrate that $s_2(t) \leq s_1(t)$ for $t \leq c = \frac{\log\frac{5}{3}}{2\pi^2}$, then we have shown that $f(t) \leq s_1(t)$ for all $t > 0$.

Thanks to Rani Moran for signalling this mistake. Many thanks to Ronald Lichters, Wiebe Pestman and Tim Loossens for their help with studying the problem, but my greatest gratitude goes to Christoph Klauer for providing the proof and Raphael Hartmann for checking it.

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✉ Francis Tuerlinckx
francis.tuerlinckx@kuleuven.be

[1] Faculty of Psychology and Educational Sciences, KU Leuven - University of Leuven, Leuven, Belgium

Now $s_2(t) \leq s_1(t)$ is equivalent to $s_1(t) \geq s_2(t)$ and thus to:

$$\frac{\pi}{4} e^{-\frac{\pi^2 t}{8}} \geq \frac{1}{\sqrt{2\pi}} t^{-\frac{3}{2}} e^{-\frac{1}{2t}}.$$

Because the left-hand term is monotonically decreasing in t, it is sufficient to shown for $t \leq c$ that:

$$\frac{\pi}{4} e^{-\frac{\pi^2 c}{8}} \geq \frac{1}{\sqrt{2\pi}} t^{-\frac{3}{2}} e^{-\frac{1}{2t}}.$$

Multiplying both sides by $\sqrt{2\pi}$ gives:

$$\left(\frac{\pi}{2}\right)^{\frac{3}{2}} e^{-\frac{\pi^2 c}{8}} \geq t^{-\frac{3}{2}} e^{-\frac{1}{2t}}.$$

Let the (constant) left-hand term be denoted by α. Because both the left-hand and the right-hand terms are positive, we can raise both to the $\frac{2}{3}$ power, giving:

$$\alpha^{\frac{2}{3}} \geq t^{-1} e^{-\frac{1}{3t}}. \tag{1}$$

Let $x = \frac{1}{t}$, then Eq. 1 is equivalent to:

$$\alpha^{\frac{2}{3}} \geq x e^{-\frac{1}{3}x},$$

or

$$-\frac{\alpha^{\frac{2}{3}}}{3} \leq -\frac{1}{3} x e^{-\frac{1}{3}x}.$$

It is easy to show that the function $w \to -we^{-w}$ has a global minimum at $w = 1$ with value $e^{-1} \approx -0.3678794$. Since $-\frac{\alpha^{\frac{2}{3}}}{3} \approx -0.5125721$, we have that:

$$-\frac{\alpha^{\frac{3}{2}}}{3} \leq e^{-1} \leq -\frac{1}{3} x e^{-\frac{1}{3}x}.$$

It thus follows that $s_2(t) \leq s_1(t)$ for $t \leq c$. Therefore, we have proven that

$$f(t) \leq s_1(t) = M\lambda e^{-\lambda t} = \frac{\pi(e^{\mu} + e^{-\mu})}{4} e^{-\frac{(4\mu^2 + \pi^2)t}{8}} \quad \text{for } t > 0.$$

As indicated in Drugowitsch (2016), this result can be generalized to a symmetric Wiener process with an arbitrary boundary separation a and variance σ^2. Thus, the crucial inequality on which the rejection algorithm rests is correct.

References

Drugowitsch, J. (2016). Fast and accurate Monte Carlo sampling of first-passage times from Wiener diffusion models. *Scientific Reports, 6*(1), 1–13.

Tuerlinckx, F., Maris, E., Ratcliff, R., & De Boeck, P. (2001). A comparison of four methods for simulating the diffusion process. *Behavior Research Methods, Instruments, & Computers, 33*(4), 443–456.

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