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Bayesian Data Analysis with the Bivariate Hierarchical Ornstein-Uhlenbeck Process Model

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ABSTRACT

In this paper, we propose a multilevel process modeling approach to describing individual differences in within-person changes over time. To characterize changes within an individual, repeated measures over time are modeled in terms of three person-specific parameters: a baseline level, intraindividual variation around the baseline, and regulatory mechanisms adjusting toward baseline. Variation due to measurement error is separated from meaningful intraindividual variation. The proposed model allows for the simultaneous analysis of longitudinal measurements of two linked variables (bivariate longitudinal modeling) and captures their relationship via two person-specific parameters. Relationships between explanatory variables and model parameters can be studied in a one-stage analysis, meaning that model parameters and regression coefficients are estimated simultaneously. Mathematical details of the approach, including a description of the core process model—the Ornstein-Uhlenbeck model—are provided. We also describe a user friendly, freely accessible software program that provides a straightforward graphical interface to carry out parameter estimation and inference. The proposed approach is illustrated by analyzing data collected via self-reports on affective states.

KEYWORDS

Intensive longitudinal data analysis; dynamical modeling; Ornstein-Uhlenbeck; Bayesian modeling; individual differences

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Introduction

Recent advances in social science data collection strategies have led to a proliferation of data sets that consist of long chains of longitudinal measurements taken from different persons. For example, the widely used methods of experience sampling (Csikszentmihalyi & Larson, 1987), or the more general ecological momentary assessments (Stone & Shiffman, 1994), provide researchers with a wide variety of measurements in natural settings. Such data often require complex statistical analyses. A new field, called *intensive longitudinal data analysis* (ILD; see, e.g., Mehl & Conner, 2012; Walls & Schafer, 2006) has emerged to meet this demand. Its strategies focus on analyzing temporal data of several participants with an emphasis on capturing interindividual variations in terms of parameters describing intraindividual variability. Unpacking underlying characteristics and processes related to intraindividual variability has crucial importance in many domains, including developmental research (see, e.g., Ram & Gerstorf, 2009), personality psychology, and emotion research (see, e.g., Kuppens, Oravecz, & Tuerlinckx, 2010; Kuppens, Stouten, & Mesquita, 2009).

We propose characterizing longitudinal measurements from an individual in terms of parameters of the Ornstein-Uhlenbeck (OU; Oravecz & Tuerlinckx, 2011; Oravecz, Tuerlinckx, & Vandekerckhove, 2011; Uhlenbeck & Ornstein, 1930) process. Using a univariate OU process to model within-person change over time in one longitudinal variable enables us to describe dynamic characteristics such as intraindividual variation and dynamic stability maintenance processes, such as regulation and adaptation. Extending this framework to two longitudinal variables within person, a bivariate OU process can additionally capture coupled within-person variation in terms of cross-effect parameters, such as covariation.

Consider an experience-sampling study that aims to study affective instability (see, e.g., the study described in the Application section). In such studies, participants (commonly more than 20) are semirandomly prompted, for example, through a mobile app to report their levels of arousal (activation) and valence (pleasantness) at the moment of the prompt in the midst of their daily activities. This integral blend of pleasure and arousal is often labeled *core affect* (Russell, 2003) in the emotion literature, and change over time in terms of self-reported core affect has been the focus of several studies (see, e.g.,

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Barrett, 2004; Kuppens, Tuerlinckx, Russell, & Barrett, 2013). Recently, neural correlates of core affect have also been found (Wilson-Mendenhall, Barrett, & Barsalou, 2013).

The resulting data set would naturally have different numbers of observations per participant, taken at different timepoints (unbalanced, unequally spaced data). Commonly used models to analyze these data often resort to models that assume equal spacing of measurements; for example, discrete time series models (Bolger & Laurenceau, 2013; Walls & Schafer, 2006). Not only can discretization bias inference (Delsing, Oud, & Bruyn, 2005), but it turns out that unequally sampling designs might be more advantageous when the sampling rate is low (Voelkle & Oud, 2013), which might occur in experience-sampling studies when researchers try to decrease burden on the participants. The proposed OU model assumes that the underlying change mechanisms behind the core affect self-reports take place in continuous time, and the unequally spaced and unbalanced observed data are samples from this process; therefore, it is especially well fit for analyzing intensive longitudinal data from experience-sampling studies.

The latent OU process we propose for modeling observed data can be characterized in terms of the following parameters: Each participant can be described with a baseline (which is a baseline core affect in our example) and regulatory mechanisms with different levels of intensity to regulate toward this baseline. Around this baseline people exhibit different levels of intraindividual variation, which in the proposed model is separated from measurement error variation through state space modeling (Fahrmeir & Tutz, 2001). That is to say that the OU framework allows us to decompose manifest variability in the raw self-reports of pleasantness and activation scores into psychologically meaningful parameters. *Psychologically meaningful* means that our model parameters can be interpreted as directly representing psychological concepts such as intraindividual affect variation and regulation. Moreover, synchronicity in changes between activation and pleasantness levels along with concurrence in regulatory dynamics are captured through two person-specific model parameters. The baseline, variation, regulatory mechanisms, and synchronicity are parameters of the OU model and can be considered as meaningful indicators of affective system quality.

In the proposed OU framework, all of these indicators describing the within-person change can be made a function of time-invariant covariates (TICs). These can be any explanatory variables that are considered relatively stable over time. For example, we hypothesized that a person's tendency for rumination (for its measurement, see Trapnell & Campbell, 1999) might be connected to the self-regulation parameter of our model; therefore, we regressed this parameter (and other model parameters as well) on this covariate. Moreover, the baseline levels of pleasantness and activation for each individual can be adjusted as a function of time-varying covariates (TVC), therefore allowing for example adaptation mechanisms to enter into the model. For example, actual measurement time can be turned into a TVC to investigate whether (and how) the baseline changes over time. To conclude, the proposed framework enables the researcher to approach a multifaceted substantive problem with a realistic and necessary level of complexity.

The proposed model also carries several desirable characteristics from the statistical inference perspective. Most important, inference in all parameters is performed in a single step. Traditional approaches (such as the maximum likelihood framework) routinely derive point estimates for parameters (e.g., for intraindividual variance) and then in a second step link these point estimates to covariates (e.g., regressing intraindividual variation in affect on neuroticism scores). This approach is problematic as relying on point estimates neglects the error that is in the parameter estimates. In the one-step approach, process model parameters, regression terms, and error terms are all estimated simultaneously, providing a principled way of propagating error in the parameter estimates. Finally, implementing parameter estimation in the Bayesian statistical framework results in probability distributions for each model parameter. This allows us to evaluate likely values of the model parameters in probabilistic terms, such as how likely it is that a parameter is larger than 0 or that it is within a certain range.

The Ornstein-Uhlenbeck process model shows correspondence to other modeling techniques often utilized for modeling for ILD. It is similar to the traditional bivariate linear mixed models (LMM; see, e.g., MacCallum, Kim, Malarkey, & Kiecolt-Glaser, 1997) in the sense that the mean structure (baseline for OU) can be made a function of TICs and TVCs. However, there are several distinctions as well. First, the proposed bivariate OU model assumes that dynamics occur on the latent level; therefore, the intraindividual variation and autocorrelation structure are modeled on the latent level. Moreover, whereas an autoregressive error term can be added to LMMs, they are typically not allowed to vary across individuals nor can they be made a function of covariates. The OU model's self-regulation parameter controls for the autocorrelation in the changes and can be made person specific. Besides the correspondence with the LMM framework, the OU model falls into the class of dynamical models termed as stochastic differential equations (SDEs; see, e.g., Chow, Ferrer, & Nesselroade, 2007; Molenaar & Newell, 2003; Oud, 2007; Oud & Jansen, 2000), which extend ordinary differential equations (ODEs; e.g., the oscillator model: Chow, Ram, Boker, Fujita, & Clore, 2005; the reservoir

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model: Deboeck & Bergeman, 2013) in allowing for process noises or uncertainties in how the latent processes change over time. When compared to oscillatory models of change, the OU model does not reinforce an oscillatory pattern on the dynamics itself but can include cyclic changes in baseline levels (as TVCs), while at the same time capturing stochastic variation that is separate from measurement noise. The proposed modeling framework also extends other current SDE models in the psychometric literature or their discrete-time difference equation and time series counterparts (e.g., Browne & Nesselroade, 2005; Chow, Ho, Hamaker, & Dolan, 2010; Wang, Hamaker, & Bergeman, 2012) by also allowing for random effects, or between-person variations, in terms of meaningful process model parameters (baseline, intraindividual variation, self-regulation, and synchronicity), as well as ways to incorporate the effects of time-varying covariates on these parameters.

Besides expounding on the technical account of the proposed model and illustrating its advantages, we aim to provide guidelines on how model fitting can be done in a practical sense. This stems from the recognition that our proposed approach does not represent the mainstream of methods used in the field of intensive longitudinal data analysis. We introduce the basic notions of Bayesian data analysis, including posterior predictive model checks. The research tool to carry out inference, the Bayesian hierarchical Ornstein-Uhlenbeck modeling (BHOUM) program will also be discussed. BHOUM is a user-friendly parameter estimation engine with a graphical user interface. It is a stand-alone program and can be downloaded (optionally with its MATLAB source code) from the first author's website (www.zitaoravecz.net). The focus of this article is on carrying out the data analysis and the interpretation of the results, and we further refer prospective users to the detailed User's Guide on the BHOUM software (available on the journal's website as supplemental material).

Investigating temporal dynamics in terms of process model parameters has potential applications in many areas. One example is core affect, defined above. Kuppens et al. (2010) formulated the *DynAffect* theory that linked general characteristics of core affect changes to Ornstein-Uhlenbeck process parameters such as attraction point or baseline, intraindividual variation, and regulatory force. We chose the DynAffect framework to demonstrate data analysis with the hierarchical OU model, and we will reanalyze data from Kuppens et al. (2010), Study 1. Our approach goes several steps further than the original analysis as we will introduce time-varying and time-invariant covariates in a one-stage analysis. Moreover, we study the cross-effect parameters of the two dimensions of core affect.

The bivariate hierarchical Ornstein-Uhlenbeck model

Model specification: The bivariate Ornstein-Uhlenbeck state space model

The core of the proposed model is the Ornstein-Uhlenbeck stochastic process, first described by two Dutch scientists Leonard Ornstein and George Eugene Uhlenbeck (Uhlenbeck & Ornstein, 1930). The OU process characterizes the within-person dynamics on the latent level; for example, the underlying changes in one's core affect. The process can be seen as a continuous time analogue of a discrete-time first-order autoregressive (AR1) process. In an AR1 process we regress the current position of the process on its previous position one time unit earlier. Similarly, the current position of the OU process depends on its previous position, but instead of one time unit of difference, the elapsed time between the two positions can take any positive value. This idea is mathematically formalized by a differential equation, which describes the rate of change in the process level over any chosen amount of time. The OU process is also perturbed by some noise; therefore, its mathematical formula is a stochastic differential equation, defined as follows:

$$d\boldsymbol{\Theta}(t) = \mathbf{B}(\boldsymbol{\mu} - \boldsymbol{\Theta}(t))dt + \boldsymbol{\Sigma}d\mathbf{W}(t) \qquad (1)$$

Equation (1) is referred to as the "dynamics (or state) equation" in the state-space modeling framework. Let us expound on Equation (1): $\boldsymbol{\Theta}(t)$ (2×1) is a two-dimensional latent random variable; for example, levels of pleasantness and activation at time t; $d\boldsymbol{\Theta}(t)$ (2×1) represents the change in these levels with respect to time t, and the right side of the equation shows that this is partly determined by the distance between the current position of the process $\boldsymbol{\Theta}(t)$ (2×1), from the baseline, denoted by $\boldsymbol{\mu}$ (2×1).[1] The level of self-regulation is expressed through the 2×2 regulatory force (or drift or dampening) matrix, $\mathbf{B}$. The other factor governing change in the latent process is the second term on the right side of Equation (1), $\boldsymbol{\Sigma}d\mathbf{W}(t)$, which represents the stochastic component of the process. The term $\mathbf{W}(t)$ stands for the position of a Wiener process (also known as Brownian motion) at time t: this process evolves in continuous time, and its position is uninfluenced by its previous positions, meaning that it follows a random trajectory. Practically speaking, the $d\mathbf{W}(t)$ term adds random variation (noise) to the system. Finally, the effect of this is scaled by the diffusion matrix $\boldsymbol{\Sigma}$ (2×2) (details follow).

[1] As a result of the mean-reverting specification (as shown in Equation [1]) the OU process does not have an ever-expanding variance expectation as in basic random walk processes.

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Integrating over the transition equation, Equation (1), results in the position equation:

$$\boldsymbol{\Theta}(t) = e^{-\mathbf{B}m}\boldsymbol{\Theta}(t-m) + \boldsymbol{\mu}(1-e^{-\mathbf{B}m}) + \boldsymbol{\Sigma}e^{-\mathbf{B}m}\int_{t-m}^{t} e^{\mathbf{B}u}d\mathbf{W}(u),$$

which shows the position of the process after elapsed time m. The last term on the left-hand side is a so-called stochastic integral. Stochastic integrals cannot be solved by regular calculus, but require special approaches, such as the Ito calculus. The Ito calculus extends the methods of regular calculus to the domain of stochastic processes. The solution in our case (Dunn & Gipson, 1977) leads to the following equation:

$$\boldsymbol{\Theta}(t+m) \mid \boldsymbol{\Theta}(t) \sim N_2(\boldsymbol{\mu} + e^{-\mathbf{B}m}(\boldsymbol{\Theta}(t)-\boldsymbol{\mu}), \boldsymbol{\Gamma} - e^{-\mathbf{B}m}\boldsymbol{\Gamma}e^{-\mathbf{B}^{\mathrm{T}}m}), \quad (2)$$

resulting in an equation that describes the conditional distribution of the position of the process after elapsed time m. Notation ^{T} stands for transpose of a matrix.

Equations (2) and (3) additionally feature the matrix $\boldsymbol{\Gamma}$, which is the stationary covariance matrix of the process—that is, the variance of the process run for an infinitely long time. $\boldsymbol{\Gamma}$ is related to the diffusion matrix $\boldsymbol{\Sigma}$ and drift matrix $\mathbf{B}$ through the following equation (see, e.g., Gardiner, 1986, p. 110):

$$\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{\mathrm{T}} = \mathbf{B}\boldsymbol{\Gamma} + \boldsymbol{\Gamma}\mathbf{B}^{\mathrm{T}}. \quad (3)$$

Equation (3) demonstrates that the scale of the diffusion process can be partitioned into a dampening contribution of the mean-reversion process (governed by the regulatory force matrix $\mathbf{B}$) and the stationary covariance. This is particularly useful since this reparameterization allows us to express the process in terms of psychologically meaningful parameters. Finally, coupled influences are captured by the off-diagonal elements of $\boldsymbol{\Gamma}$ (covariation) and $\mathbf{B}$ (synchronicity in self-regulation).

As can be seen, over time the OU process continuously drifts toward the baseline due to the mean-reverting dynamics. In addition, there is a stochastic input term that influences the change trajectory. Psychological processes for which this type of perturbation and mean reversion can be an appropriate model include emotion, mood and affect regulation (Gross, 2002), semantic foraging (i.e., search in semantic memory; see in Hills, Jones, & Todd, 2012), and so on.

The empirical measurements, $\mathbf{Y}(t)$ (e.g., pleasantness and activation self-reports), are typically discrete. Therefore, we link the observed discrete data to the continuous underlying state (or levels) of the process by adding some measurement error. In the state space modeling framework, the equation describing this idea is called the "observation equation." We map the latent dynamics to the manifest data through the following specifications:

$$\mathbf{Y}(t) = \boldsymbol{\Theta}(t) + \boldsymbol{\epsilon}(t). \quad (4)$$

The measurement error is represented by $\boldsymbol{\epsilon}(t_s)$, which is distributed according to a bivariate normal distribution with expectation $(0, 0)^{\mathrm{T}}$ and covariance matrix $\boldsymbol{\Sigma}_\epsilon$. Next, we expand this basic state space model with a hierarchical structure to be able to fit a multilevel OU model to intensive longitudinal data.

Hierarchical extension: The bivariate multilevel OU model for intensive longitudinal data

A typical structure for an intensive longitudinal data set would be the following: Longitudinal variables for a person p $(p = 1, \ldots, P)$ are measured at n_p timepoints: $t_{p1}, t_{p2}, \ldots, t_{ps}, \ldots, t_{p,n_p}$. We jointly model two data points for each person p at timepoint t_{ps}, denoted as $\boldsymbol{Y}(t_{ps}) = (Y_1(t_{ps}), Y_2(t_{ps}))^{\mathrm{T}}$. The index s denotes the sth measurement occasion of that individual. In the hierarchical Ornstein-Uhlenbeck (HOU) model we assume that these observations are functions of a latent underlying state denoted as $\boldsymbol{\Theta}(t_{ps}) = (\Theta_1(t_{ps}), \Theta_2(t_{ps}))^{\mathrm{T}}$ and some measurement error.

As proposed, the underlying latent states (e.g., changes in one's core affect) are assumed to be governed by a two-dimensional OU process. For simplicity, we use only the indices p and s when denoting parameters or data that are related to the specific observation at t_{ps}. Then an HOU model for a single person p can be written as

$$\boldsymbol{Y}_{ps} = \boldsymbol{\Theta}_{ps} + \boldsymbol{\epsilon}_{ps}, \quad (5)$$

where $\boldsymbol{Y}_{ps}$ is a shorthand for $\boldsymbol{Y}(t_{ps})$ and stands for the observed random vector; $\boldsymbol{\Theta}_{ps}$ denotes the latent state (or true score, shorthand for $\boldsymbol{\Theta}(t_{ps})$); and $\boldsymbol{\epsilon}_{ps}$ stands for the measurement error with the distributional assumption: $\boldsymbol{\epsilon}_{ps} \overset{\text{iid}}{\sim} N_2(\mathbf{0}, \boldsymbol{\Sigma}_\epsilon)$. Based on Equation (2), the latent underlying level of the bivariate process for person p at time point s can be written as

$$\boldsymbol{\Theta}_{ps} | \boldsymbol{\Theta}_{p,s-1} \sim N_2\left(\boldsymbol{\mu}_{ps} + e^{-\mathbf{B}_p(t_{ps}-t_{p,s-1})}(\boldsymbol{\Theta}_{p,s-1} - \boldsymbol{\mu}_{ps}), \boldsymbol{\Gamma}_p - e^{-\mathbf{B}_p(t_{ps}-t_{p,s-1})}\boldsymbol{\Gamma}_p e^{-\mathbf{B}_p^{\mathrm{T}}(t_{ps}-t_{p,s-1})}\right). \quad (6)$$

Parameter $\boldsymbol{\mu}_{ps}$ is the person-specific bivariate baseline, which can also be referred to as home base, bivariate attractor, or attraction point, and is somewhat similar to a mean vector. It can change over time as a function of time-varying covariates. For example, baseline valence and arousal levels can change as function of actual time during the daily cycle. Variation around the baseline is modeled through $\boldsymbol{\Gamma}_p$, which is a person-specific intraindividual 2-by-2 covariance matrix. In terms of valence and arousal, the two diagonal elements represent variation in

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these, while the off-diagonals capture the covariation in the changes.

The model assumes that there is always some level of attraction, or regulation over time toward the baseline level, and the dynamics of this are modeled through the 2-by-2 regulatory force matrix $\mathbf{B}_p$. Following our core affect example, for each person arousal and valence levels can be regulated with different intensity, represented by the two diagonal elements of the $\mathbf{B}_p$ matrix, and the cross effect of these dynamics, the off-diagonal of $\mathbf{B}_p$, is also a person-specific parameter.

Finally, for the the first observation, $\mathbf{\Theta}_{p1}$, it is assumed that $\mathbf{\Theta}_{p1} \sim N_2(\boldsymbol{\mu}_p, \mathbf{\Gamma}_p)$. This is because Equation (2) cannot be used for the first observation as there is no previous position to condition on. Therefore, we assume that the first observation is simply a function of the person's baseline level and intraindividual variance.

To summarize, the hierarchical Ornstein-Uhlenbeck model represents a process modeling approach to modeling change in two longitudinal measures. As explained, the changes over time in the two manifest longitudinal variables are governed by an underlying latent process (the OU process). Since we also assume that changes in the two longitudinal variables are interlinked, this process model approach can be visualized by picturing a latent process evolving over time among the two dimensions, which are being defined by our two longitudinal variables of interest. The actual longitudinal measurements we collect are samples from the current position of this latent process evolving in the two-dimensional space, and these measurements are perturbed by measurement error. The OU model based dynamics on the person-level (or Level 1) are described by Equation (6). Given the hierarchical nature of our model, bivariate longitudinal data for each person is described by a unique set of OU process parameters, coming from joint Level 2 (population) distributions. Next we expound upon the parameterizations, define Level 2 distributions, and introduce time-varying and time-invariant covariates.

The two-dimensional baseline as a function of time-varying and time-invariant covariates

The latent baseline levels (or attraction point parameter) $\boldsymbol{\mu}_{ps}$ can be made a function of person-specific time-varying and person-specific time-invariant covariates. Let us assume that we measure time-invariant covariate j, for person p x_{jp}, which could be for example their tendency to ruminate. We can have k TICs measured, ($j = 1, \ldots, k$), and the TICs can be collected into a vector of length $k+1$, denoted as $\boldsymbol{x}_p = (x_{p0}, x_{p1}, x_{p2}, \ldots, x_{pk})^{\mathrm{T}}$, with $x_{p0} = 1$. Even if there is no time-invariant covariate information, we assume an intercept in the model.

Regarding the time-varying aspect, suppose that we measure covariate z for person p, and $z = 1, \ldots, E$, then the vector $\boldsymbol{z}_{ps} = (z_{ps1}, \ldots, z_{psE})^{\mathrm{T}}$ collects all these values. No intercept is introduced in the vector $\boldsymbol{z}_{ps}$. The index s indicates that values may change from one observation point to the next. A natural candidate for a TIC to regress baseline levels of valence and arousal is the time of the self-report; for example, we expect some people to show low levels of pleasantness and activation in the morning.

The Level 2 distribution (distribution on the "population" level) of $\boldsymbol{\mu}_{ps}$ with regression on the time-invariant and time-varying covariates and allowing for a person-specific random deviation can be written as follows:

$$\boldsymbol{\mu}_{ps} \sim N_2\left(\boldsymbol{\Delta}_{p\mu}\boldsymbol{z}_{ps} + \boldsymbol{A}_{\mu}\boldsymbol{x}_p, \boldsymbol{\Sigma}_{\boldsymbol{\mu}}\right), \tag{7}$$

where the covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\mu}}$ is defined as follows:

$$\boldsymbol{\Sigma}_{\boldsymbol{\mu}} = \begin{bmatrix} \sigma^2_{\mu_1} & \sigma_{\mu_1\mu_2} \\ \sigma_{\mu_1\mu_2} & \sigma^2_{\mu_2} \end{bmatrix}. \tag{8}$$

The matrices $\boldsymbol{\Delta}_{p\mu}$ and $\boldsymbol{A}_{\mu}$ are parameter matrices of dimension $2E \times P$ and $2 \times (k+1)$, respectively, containing the regression weights for the time-varying and the time-invariant covariates.

The intraindividual covariance matrix as a function of time-invariant covariates

The matrix $\mathbf{\Gamma}_p$ stands for the stochastic or intraindividual 2×2 covariance matrix

$$\mathbf{\Gamma}_p = \begin{bmatrix} \gamma_{1p} & \gamma_{12p} \\ \gamma_{12p} & \gamma_{2p} \end{bmatrix}. \tag{9}$$

Its diagonal elements (i.e., γ_{1p} and γ_{2p}) determine the intraindividual variances in the two measured longitudinal variables, and the off-diagonals can be decomposed into $\gamma_{12p} = \rho_{\gamma_p}\sqrt{\gamma_{1p}\gamma_{2p}}$, where ρ_{γ_p} is the cross correlation of the observations. Since the diagonal elements (γ_{1p} and γ_{2p}) are variances, they are constrained to be positive. For computational convenience, we log transform these variances so that they take values on the real line. Then we specify their Level 2 distributions as normal distributions of these log-transformed values. For γ_{1p}, that is

$$\log\left(\gamma_{1p}\right) \sim N\left(\boldsymbol{x}_p^{\mathrm{T}}\boldsymbol{\alpha}_{\gamma_1}, \sigma^2_{\gamma_1}\right).$$

The mean of this distribution is modeled via the product of time-invariant covariates and their corresponding regression weights. More specifically, all TICs are collected in the vector $\boldsymbol{x}_p^{\mathrm{T}}$, which has $k+1$ components. The first element of this vector is a constant 1, representing an intercept, and in case there are no TICs added, the mean of the distribution reduces to a simple Level 2 mean that is the same for all persons. The vector $\boldsymbol{\alpha}_{\gamma_1}$ contains the (fixed) regression coefficients for the covariates. The

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parameter $\sigma^2_{\gamma_1}$ is the residual variance in the random log variance of the first dimension, after having taken the covariates into account. If only the intercept is present in the model, $\sigma^2_{\gamma_1}$ reflects the total amount of interindividual variability in the log variance of the first dimension. A similar logic applies in the modeling of γ_{p2}.

The cross correlation ρ_{γ_p} is bounded between -1 and 1. By taking advantage of the Fisher z transformation $F(\rho_{\gamma_p}) = \frac{1}{2}\log\frac{1+\rho_{\gamma_p}}{1-\rho_{\gamma_p}}$, we can transform its values to the real line:

$$F(\rho_{\gamma_p}) \sim N\left(\boldsymbol{x}_p^{\mathrm{T}}\boldsymbol{\alpha}_{\rho_\gamma}, \sigma^2_{\rho_\gamma}\right),$$

with $e_{p\gamma_2} \sim N(0, \sigma^2_{\rho_\gamma})$. The density of the original ρ_{γ_p} can be derived by applying a transformation-of-variables technique (see, e.g., Mood, Graybill, & Boes, 1974), but it is not a common density function. Again, $\boldsymbol{\alpha}_{\rho_\gamma}$ contains $k+1$ regression weights; $\boldsymbol{x}_p^{\mathrm{T}}$ contains the k covariate values for person p with 1 for the intercept; and $\sigma^2_{\rho_\gamma}$ represents interindividual variation in terms of cross correlation.

The regulatory force as a function of time-invariant covariates

The regulatory force or centralizing tendency is parameterized by the matrix $\mathbf{B}_p$, which is decomposed in the same manner as the covariance matrix $\mathbf{\Gamma}_p$ in Equation (9), so that it stays positive definite. Matrix $\mathbf{B}_p$ has to stay positive definite by definition to ensure that there is always an adjustment toward the baseline and never away from it. This implies that the process is stable and stationary.

The elements of the person-specific matrix $\mathbf{B}_p$ are assumed to come from Level 2 distributions that are defined in the same manner as for $\mathbf{\Gamma}_p$ and can be made the function of time-invariant covariates in the same manner. This way $\mathbf{B}_p$ contains two centralizing tendencies, one for each dimension (i.e., β_{1p} and β_{2p}), and a standardized cross-centralizing tendency parameter (ρ_{β_p}) that represents the concurrence in regulatory dynamics. These parameters control the strength and the direction of the self-regulation toward the baseline. As β_{1p} and β_{2p} go toward zero (i.e., no self-regulation), the OU process approaches a Brownian motion process; that is, a continuous-time random walk process. When the two parameters become very large and tend toward infinity, the OU process becomes a white noise process.

Bayesian statistical inference in the HOU model

We implemented parameter estimation for the hierarchical OU model by taking advantage of Bayesian statistical methods. The Bayesian approach features two main advantages in our current settings. First, parameters in this framework have probability distributions, which offers an intuitively appealing way of describing uncertainty and knowledge about the parameters. Second, there are distinct computational advantages: the use of Markov chain Monte Carlo (MCMC) methods sidesteps the high-dimensional integration problem over the numerous random effect distributions.

When carrying out Bayesian data analysis, we use these stochastic numerical integration methods to sample from the *posterior density* of the parameters. The posterior density is the conditional density function of the parameters given the data, and it is directly proportional to the product of the likelihood of the data (given the parameters) and the prior distribution of the parameters.[2] The prior distribution incorporates prior knowledge about the parameters, and if there is none, it is can be set to a vague (diffuse) distribution. The BHOUM toolbox follows this philosophy: all priors are set to be vague. In addition, the more data one acquires, the less influential the prior becomes on the posterior as its shape is overwhelmed by the tighter shape of the likelihood.

Markov chain Monte Carlo methods are general-purpose algorithms for sampling from the high-dimensional posterior of the presented model. MCMC algorithms perform iterative sampling during which values are drawn from approximate distributions that are improved in each step, in such a way that they converge to the targeted posterior distribution. After a sufficiently large number of iterations, one obtains a Markov chain with the posterior distribution as its equilibrium distribution, and the generated samples are random draws from the posterior distribution. Summary statistics of the generated samples can then be used to characterize the posterior distribution (i.e., to estimate its mean, variance, mass over a certain interval, etc.) More details about the Bayesian methodology and MCMC can be found in Gelman, Carlin, Stern, and Rubin (2013) and Robert and Casella (2004). For the HOU model there is no closed-form analytical solution for the main parameters of interest; therefore, high-dimensional numerical integration is required to calculate posterior point estimates. With MCMC methods we can solve this problem while avoiding having to explicitly calculate $p(\boldsymbol{Y})$.

In the BHOUM toolbox, a specific MCMC algorithm—the Metropolis-within-Gibbs sampler—is implemented to estimate the HOU model parameters. In this algorithm, alternating conditional sampling is performed: The parameter vector is divided into subparts (a single element or a vector), and in each iteration the algorithm

[2] Formally, $p(\boldsymbol{\xi}|\boldsymbol{Y}) \propto p(\boldsymbol{Y}|\boldsymbol{\xi})p(\boldsymbol{\xi})$, where $\boldsymbol{\xi}$ stands for the vector of all parameters in the model. The normalization constant, $p(\boldsymbol{Y})$, where $\boldsymbol{Y}$ stands for the data, does not depend on the parameter and is therefore not considered here.

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draws a new sample from the conditional distribution of each subpart given all the other parameters and data; these conditional distributions are called *full conditional* distributions. In our application, several such Markov chains are initiated from different starting values in order to explore the posterior distribution and avoid local optima. The BHOUM toolbox offers a default convergence check using the Gelman-Rubin $\hat{R}$ statistic (for more information, see Gelman, Carlin, Stern, & Rubin, 2004).

Data: Experience-sampling study on core affect

Study settings

In this section we provide a description of how to use the BHOUM software through analyzing data from an experience-sampling study. The corresponding data set was collected at the University of Leuven (Belgium) and contains repeated measurements of 79 university students' pleasantness and activation levels (i.e., their core affect).

Per the principles of the experience-sampling design, measurements were made in the participants' natural environments: They carried a Tungsten E2 palmtop computer that was programmed to beep at semirandom times during waking hours over 14 consecutive days. When signaled by a beep, the participants were asked to mark their position on a 99×99 core affect grid with unpleasant–pleasant feelings forming the horizontal dimension and arousal–sleepiness the vertical.

Moreover, several dispositional questionnaires were administered to measure a range of covariates in the participants. These variables were neuroticism and extraversion (part of the five-factor model of personality, or Big Five; Costa & McCrae, 1992; for the current study a translated version was used, see in Hoekstra, Ormel, & De Fruyt, 1996), positive and negative affect (PA and NA; Hoeksma, Oosterlaan, Schipper, & Koot, 1988), self-esteem (and self-esteem variability; Rosenberg, 1989), satisfaction with life (Diener, Emmons, Larsen, & Griffin, 1985), reappraisal and suppression (Gross & John, 2003), and rumination (Trapnell & Campbell, 1999). These covariates were used as time-invariant covariates in the analysis that follows.

Summary of the proposed data-analytical approach

Although several HOU models were fit to the data set in Kuppens et al. (2010), none of those models involved covariates. That is to say, so far all analyses were performed in two stages: OU parameters were estimated, and correlation coefficients (in the classical sense) were calculated between the person-specific Bayesian posterior point estimates and the covariate scores from the dispositional questionnaires. In the current analysis, the latent OU parameters are regressed on the time-invariant dispositional measures described at the same time as the latent dynamical process model parameters are estimated. This way, uncertainty in the parameter estimates is directly accounted for in the results so that the analysis avoids generated regressor bias (Pagan, 1984). In addition, as part of the same analysis we incorporate time-varying covariates on the baseline, thereby further improving the accuracy of the parameter estimation.

Methods: Analyzing data with the hierarchical OU model

The BHOUM toolbox contains several functions to deal with various aspects of Bayesian statistical inference. BHOUM is primarily intended to be used as a stand-alone software program (no MATLAB license is required) through a graphical user interface (GUI).[3] While no coding is required from the user's part, all MATLAB scripts are available for download.

Parameter estimation

In the current analysis we model pleasantness and activation levels of 79 people from the described experience-sampling study with a hierarchical OU process. All latent process parameters (baseline, intraindividual variation, regulation, cross effects) are modeled as functions of 10 time-invariant covariates: neuroticism, extraversion, positive affect, negative affect, self-esteem, within-person standard deviation of self-esteem, satisfaction with life, reappraisal, suppression, and rumination. Moreover, circadian rhythm in the core affect baseline is modeled in terms of linear and quadratic time effects.

Running *BHOUMtoolbox.exe* displays a user-friendly *Data reader* GUI that allows the researcher to load the data and specify which variables are chosen to be part of the analysis. For the data format needed to use the BHOUM program, please consult the supplementary online files.

Once the required data have been input, the user can move to the next window (*Model specifier*) where model and sampling algorithm specifications can be set. The default model is the one described in the previous section. In this fully specified model, all process model parameters are random effects. This way, the means of the two dimensions (μ_{1p} and μ_{2p}), the corresponding stochastic

[3] The stand-alone BHOUM version with the accompanying free MATLAB Compiler Runtime (MCR) has been tested for Windows 32bit and 64bit. If users do not want to install MCR because they have a MATLAB license already, that MATLAB should be run in 32bit mode.

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variances (γ_{1p} and γ_{2p}), and the cross correlation (ρ_{γ_p}), as well as the two regulatory force (or centralizing tendencies: β_{1p} and β_{2p}) parameters and their cross effect (ρ_{β_p}) are allowed to be person-specific and regressed on time-invariant covariates if any was previously loaded and selected in the *Data reader* window. Alternative models are offered as well, which are simplified versions of the default fully person-specific HOU model. For example, the measurement error can be removed from the model. Another option is assuming that the changes in two dimensions are independent; that is, all ρ_{γ_p} and ρ_{β_p} are equal to 0. By using this option we can also model only one longitudinal variable by inputting the same variable twice (i.e., choosing twice the same column name in Data reader, OU process dimensions) and then choosing the "Independent dimensions" option in the Model specifier. The program will then fit two independent one-dimensional HOU models.

The *Model specifier* window allows setting the properties of the Markov chain Monte Carlo sampling algorithm. Some default values are preset, and these will provide sufficient exploration of the posterior distribution in most cases. The properties are the following: (1) number of posterior samples (per chain, same for each chain), used for posterior inference, (2) length of some necessary adaptive period (the *burn-in*) preceding the samples set in box (1), (3) the number of chains that are run from different starting values to explore the posterior density, and (4) thinning factor. The thinning option is primarily implemented for computer memory capacity considerations. Because of high within-chain autocorrelation, some parameter estimates might require long chains to be run to explore the posterior density. By thinning these long chains, we store only every xth value, where x equals the input of the *Thin* field.

It is good practice to report the setting of these for values when reporting the results of the analysis. For the current analysis, we set four chains each consisting of 3,000 iterations thinned by factor 3, following an adaptation period of 2,000 iterations resulting in a final total of 12,000 posterior samples ($4 \times 3{,}000$) for each parameter. We also enabled the option to calculate the Deviance Information Criterion (DIC; Spiegelhalter, Best, Carlin, & Linde, 2002), for use in later model comparison.

When the iterations are finished, two new windows pop up: the *Result browser* and a noninteractive table that gives a summary of the posterior statistics of the most important parameters. This window shows the posterior means, standard deviations, and percentiles of these parameters. Moreover, it provides information about convergence by displaying $\hat{R}$ statistics (Gelman et al., 2004), effective number of samples (the number of independent samples, computed by using the total number of posterior samples and a measure of their mutual dependence where more dependent samples count as fewer, while entirely independent samples count fully), and sample sizes.

The *Result browser* window offers several ways to explore the results. By default, the interface shows a warning if convergence is not reached for all parameters in terms of $any(\hat{R}) > 1.1$, and graphical tools are included to explore the posterior samples of the parameters. For the current analysis, the MCMC procedure converged with all $\hat{R}$ statistics lower than 1.1.

Posterior predictive checks

Moreover, there are two posterior predictive checks (PPCs) implemented in the program. Both of them are based on generating new data sets based on the full posterior distribution of the parameters and comparing certain properties of the observed and generated data sets. The first check assesses the similarity between the observed and replicated trajectories: It computes the degree of overlap between the observed and simulated trajectories by calculating the correlation between the frequencies with which the observed data fall in a certain area in a two-dimensional space and the average frequency with which they fall in that area across replicated data sets. The resulting measure is a correlation coefficient averaged over participants. The correlation was 0.86 for the current data set, slightly improved fit compared to the same measure (0.80) reported in Kuppens et al. (2010), in the analysis without covariates.

The second PPC indicates whether the observed and replicated trajectories are similar in terms of turning angles. A turning angle is a clockwise angle between two line segments that connect three subsequent points in time in the two-dimensional space created by the two longitudinally measured variables. We average over all turning angles person-wise, resulting in a person-specific average turning angle value. We calculate this measure for replicated data sets, and based on these, a 95% prediction interval is established for every person. The program returns which proportion of the observed average turning angles falls within this interval. With respect to this measure, the current analysis showed that 95% of the generated person-specific average turning angles fell within the 95% prediction interval, showing an adequate fit of the HOU model (for more details on posterior predictive checks, see Gelman, Meng, & Stern, 1996).

Results

General characteristics

Table 1 shows Level 2 results in terms of posterior mean estimates and 95% posterior credible intervals (PCIs).

Table 1. Summary of the results from the hierarchical Ornstein-Uhlenbeck model on Level 2.

Model parameter	Description	Posterior mean	95% PCI	
		Valence		
α_{μ_1}	Baseline	5.7833	5.6097	5.9537
$\sigma^2_{\mu_1}$	Inter-individual variation in baseline	0.5270	0.3535	0.7630
$\delta_{L\mu_1}$	Linear time-effect	− 0.0871	− 0.2026	0.0225
$\delta_{Q\mu_1}$	Quadratic time-effect	0.0043	0.0008	0.0080
$e(\alpha_{\gamma_1})$	Intra-individual variance	2.9777	2.4637	3.5089
$e(\alpha_{\beta_1})$	Self-regulation	1.8535	1.4005	2.5695
$\sigma^2_{1\epsilon}$	Measurement error	0.2197	0.1485	0.2883
		Arousal		
α_{μ_2}	Baseline	4.4786	4.2594	4.6973
$\sigma^2_{\mu_2}$	Inter-individual variation in baseline	0.8212	0.5580	1.1889
$\delta_{L\mu_2}$	Linear time-effect	0.9334	0.7343	1.1351
$\delta_{Q\mu_2}$	Quadratic time-effect	− 0.0303	− 0.0366	− 0.0241
$e(\alpha_{\gamma_2})$	Intra-individual variance	3.8756	3.3555	4.5051
$e(\alpha_{\beta_2})$	Self-regulation	1.8113	1.3931	2.4668
$\sigma^2_{2\epsilon}$	Measurement error	0.4646	0.3436	0.5826
		Cross effects		
$\sigma_{\mu_1\mu_2}$	Covariance between the baseline levels	− 0.2678	− 0.4813	− 0.0943
α_{ρ_γ}	Cross-correlation	− 0.2342	− 0.3056	− 0.1621
α_{ρ_β}	Self-regulation correlation	− 0.0237	− 0.1197	0.0965

Note. The $e(.)$ stands for the expected value of that parameter on the normal scale (these parameters were estimated using the log scale).
The first column displays the mathematical notation of the parameter; the second column describes the function of the parameter. The third column displays the mean of the posterior distribution corresponding to the parameter; columns four and five summarize the lower and upper limits of a symmetric 95% posterior credibility interval (PCI).

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As can be seen, the baseline core affect is rather pleasant ($\alpha_{\mu_1} = 5.7833$) and not particularly aroused ($\alpha_{\mu_1} = 4.4786$ on a measurement scale that ranged from 0.1 to 9.99). Note that this baseline point was allowed to change as a function of measurement time nested in the diurnal cycle, specifically in terms of linear and quadratic time-varying covariates centered around noon, meaning that we allow for each person's attraction point to vary with the time of day. The average pleasantness and activation feelings in this analysis correspond to the baseline core affect *at noon*. The black lines in the two panels of Figure 1 represent the average (across persons) diurnal pattern, based on the posterior mean estimates of the linear and quadratic time effects in the valence and in the arousal dimensions. For the valence dimension, the linear time effect has a very low magnitude ($\delta_{L\mu_1} = -0.0871$), and its 95% PCI is rather wide, meaning that the valence baseline did not change as a linear function of time of day. However, there was a small quadratic time effect ($\delta_{Q\mu_1} = 0.0043$) with a comparatively narrow PCI that suggests that on average there was a small quadratic trend in the valence baseline position, which is somewhat noticeable on the black line in Figure 1, left panel. With respect to the black line in the right panel, there is a more remarkable quadratic trend in the Level 2 mean *arousal* change over time. Indeed, with respect to arousal, both linear and quadratic effects have relatively large magnitudes ($\delta_{L\mu_2} = 0.9334$, $\delta_{Q\mu_2} = -0.0303$) with comparatively narrow PCIs, (0.7343, 1.1351) and (−0.0366, −0.0241), respectively. In both plots, the gray lines correspond to the person-specific diurnal profiles in the baseline levels. There appears to be large variation in these profiles, especially in terms of intercept, and there seems to be more between-person variability with respect to arousal ($\sigma^2_{\mu_2} = 0.8212$) than with respect to valence ($\sigma^2_{\mu_1} = 0.5270$).

The average intraindividual variance was higher in the arousal dimension ($\alpha_{\gamma_2} = 3.8756$) than in the valence ($\alpha_{\gamma_1} = 2.9777$) dimension. The intraindividual variances are large compared to the measurement errors ($\sigma^2_{1\epsilon} = 0.2197$, $\sigma^2_{2\epsilon} = 0.4646$), showing that the latent process explains a large part of the variation in the data—people's movements through affect space appear to be well described by an OU process.

The magnitude of the regulatory force is larger in the arousal than in the valence dimension. This means that on average, people return to their baseline faster when their arousal level fluctuates than when their valence does. This is especially interesting since the results described also show that there is more variability in the arousal dimension. Together, these findings demonstrate that these two dynamical aspects are distinct, and more variation in the observed data does not necessarily mean the lack of self-regulation. Also, there was no evidence that regulatory dynamics in valence and arousal are systematically related to one another, as α_{ρ_β} was practically zero.

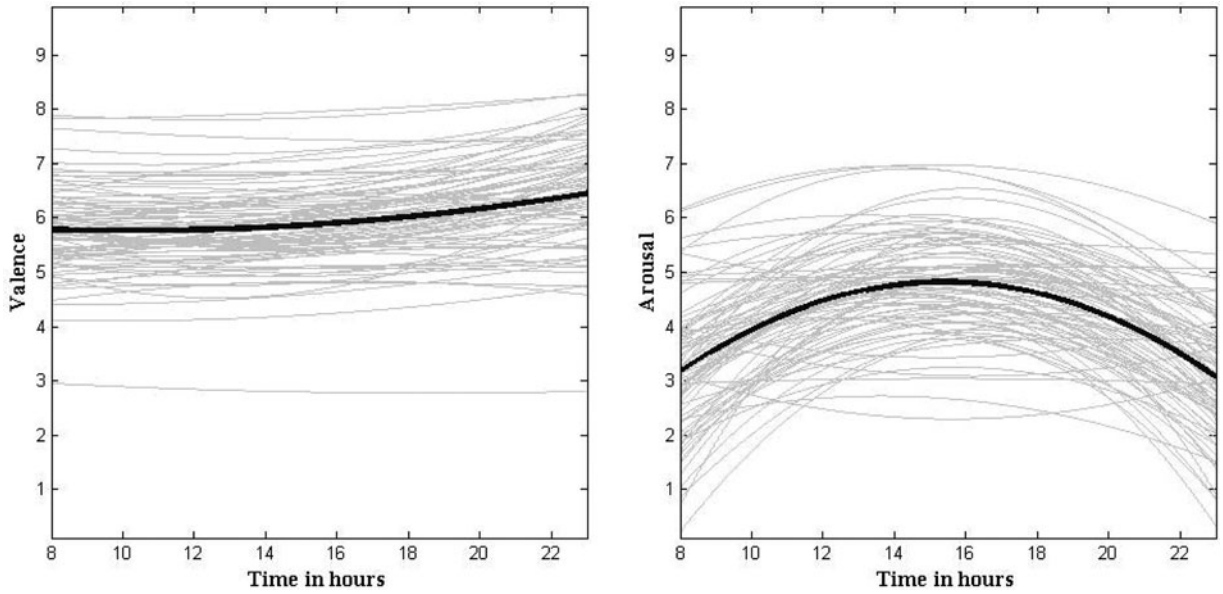

Figure 1. Daily patterns in baseline core affect dimensions. The left panel shows the pleasantness, the right panel the activation dimensions. The grey lines correspond to expected person-specific trajectories based on OU model parameter estimates. The black lines show the expected Level 1 (population) trajectory based on the corresponding parameter estimates in the two core affect dimensions.

The current analysis revealed links between the instantaneous changes in valence and arousal (average cross correlation $\alpha_{\rho_\gamma} = -0.2342$). This indicates that changes in the valence dimensions were likely accompanied with changes in the arousal dimension, in the opposite direction, and vice versa. This finding suggests, for example, that when people were aroused, their valence was likely to drop slightly. A somewhat related effect was shown by the covariance between the baseline levels: $\sigma_{\mu_1\mu_2} = -0.2678$ indicated that people who had higher arousal baseline tended to have lower valence baseline and vice versa.

Finally, Figure 2 provides yet another example of the evaluation of the model fit (see also posterior predictive checks before). While a stochastic model such as the OU model cannot be expected to fit the data perfectly, the primary utility of a process model lies in its ability to capture and quantify those qualitative aspects of the data that are relevant for psychological interpretation. The figure shows data from two participants (on the left) and four sets of model-generated data for each (on the right). In both cases, and in general for our participants, the data generated by the model strongly resemble the observed data.

Results on time-invariant covariates

All eight person-specific process model parameters were regressed on 10 time-invariant covariates; that is, a multiple regression model with 10 predictors was fitted for each of the eight parameters.[4] Based on the posterior samples, Table 2 displays the result on the regression coefficients whose 95% PCI did not contain 0: These were the effects for which the magnitude was relatively high and the corresponding 95% PCIs were comparatively narrow, providing substantial evidence that the latent process parameters differed markedly as a function of these covariates.

As expected, positive affect was positively related to the valence baseline point: People who frequently experienced positive affect tended to feel more pleasant on average. However, with respect to the baseline, there was only one more remarkable covariate: The lack of rumination strategy for controlling emotional experience predicted a more aroused baseline level.

With respect to intraindividual core affect variation, only the within-person variability in the measurement of self-esteem showed a marked effect:[5] people with more variable self-esteem had higher levels of variation in their core affect in general. This way, an important cognitive/evaluative aspect (how one thinks of oneself) was connected to affect variation.

Possibly the most compelling aspect of HOU model analysis concerns the regulatory mechanism and the cross effects. We would like to point out that self-regulation in the model refers to a stronger mean-reverting tendency.

[4] Given the simultaneous inclusion of the predictors, the estimated regression weights of each predictor here are conditional on the other predictors in the model (i.e., they indicate effects over and above those of the other predictors.)

[5] Note that, while the self-esteem measure was collected at every measurement occasion, the person-specific variance in self-esteem is time-invariant.

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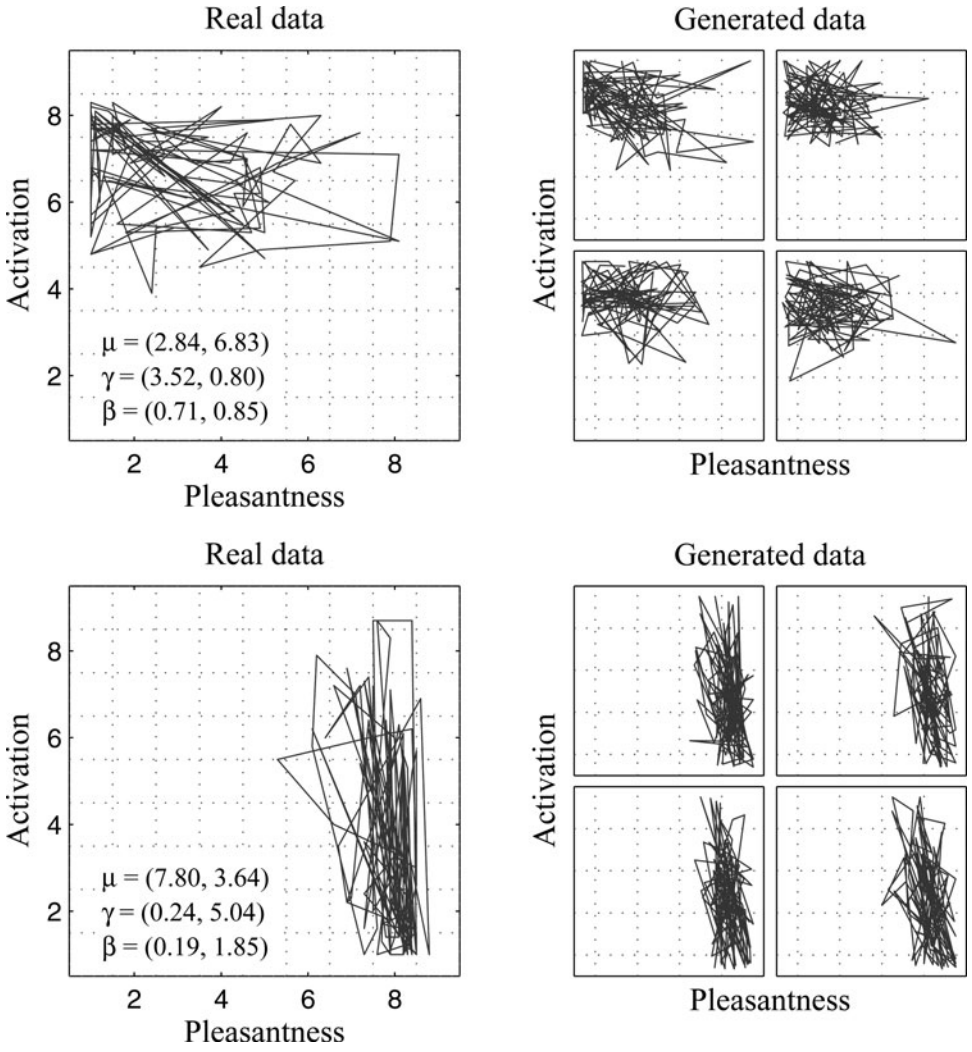

Figure 2. An illustration of the qualitative aspects of our data that are captured by the Ornstein-Uhlenbeck model. In the top left, the data from one participant are plotted. This participant has a home base in an area of low pleasantness but high activation (i.e., upset/distress), has medium variation in pleasantness but is stable in activation, and has medium levels of self-regulation. Each of the four panels in the top right contain data generated from the model using this participant's parameters. The data in the bottom left are from a participant with a home base in an area of high pleasantness and medium activation. Volatility is low in pleasantness but high in activation, and self-regulation is average. In both the top row and the bottom row, and in general for our participants, the model recreations of participant data well capture the salient qualities of the real data. In the two panels on the left side, the numbers next to μ (baseline), γ (intraindividual variation), and β (regulation) correspond to the point estimates in pleasantness and in the activation dimensions with respect to these parameters.

That is to say that its desirability might depend on the actual baseline level. As can be seen from Table 2, most of the credible effects actually relate to these aspects. First, people with higher neuroticism scores showed lower levels of valence self-regulation, while extroverts had lower levels of arousal self-regulation. Higher negative affect and self-esteem variability scores predicted better self-regulation of core affect. This suggests that people who frequently experienced negative emotions and fluctuations while reflecting on their own worth, showed higher levels of affect regulation. This brings up the question whether pathologies that are associated with negative affect and self-doubting might have underlying dynamical characteristics where negative baseline associated with strong self-regulation lead to pathological consequences. While this is only theoretical at this point as the current study is not conclusive (NA and self-esteem variability did not show remarkable association in this study), it appears to be a promising question for further exploration. Finally, from the three emotion self-regulation strategies (reappraisal, thought suppression, and rumination), only reappraisal predicted better self-regulation, and only in the arousal dimension. In fact, the other two strategies (thought suppression and rumination) are considered to be maladaptive when it comes to emotion self-regulation (see, e.g., thought suppression in Wegner

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Table 2. Summary of the regression weights with a 95% posterior credible interval not containing 0.

Model parameter	Displayed name in BHOUM	Description	Covariate	Posterior mean	95% Posterior credible interval	
			Valence			
α_{μ_1}PA	alpha_Mu_4	Attractor	Positive affect	0.24	0.01	0.47
α_{γ_1}SESD	alpha_gamma1_7	Variation	Self-esteem variability	0.28	0.11	0.46
α_{β_1}N	alpha_beta1_2	Self-regulation	Neuroticism	−0.59	−1.08	−0.12
α_{β_1}NA	alpha_beta1_5	Self-regulation	Negative affect	0.32	0.03	0.64
α_{β_1}SESD	alpha_beta1_7	Self-regulation	Self-esteem variability	0.41	0.15	0.67
			Arousal			
α_{μ_2}RUM	alpha_Mu_22	Attractor	Rumination	−0.37	−0.66	−0.09
α_{γ_2}SESD	alpha_gamma2_7	Variation	Self-esteem variability	0.20	0.05	0.36
α_{β_2}E	alpha_beta2_3	Self-regulation	Extraversion	−0.36	−0.65	−0.08
α_{β_2}NA	alpha_beta2_5	Self-regulation	Negative affect	0.30	0.02	0.59
α_{β_2}SESD	alpha_beta2_7	Self-regulation	Self-esteem variability	0.32	0.09	0.57
α_{β_2}RE	alpha_beta2_9	Self-regulation	Reappraisal	0.36	0.14	0.58
			Cross effects			
α_{ρ_γ}RUM	alpha_rho_gamma_11	Cross correlation	Rumination	0.10	0.00	0.19

Note. Model parameters refer to the regression weights. For example, α_{μ_1}PA is the regression weight for positive affect relating to the valence baseline (μ_1).
The first column displays the mathematical notation of the OU parameter; the second column shows the way they are displayed in the Bayesian hierarchical Ornstein-Uhlenbeck modeling (BHOUM) toolbox's output window. The third and fourth columns contain verbal descriptions of which model parameter was regressed on which explanatory variable. The fifth column displays the mean of the posterior distribution corresponding to the parameter; columns 6 and 7 summarize the lower and upper limits of a symmetric 95% posterior credibility interval (PCI).

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& Zanakos, 1994, and rumination in Nolen-Hoeksema, 2000).

It is interesting to note the discrepancies between our current results and those obtained from the original two-stage analysis reported in Kuppens et al. (2010). The most striking differences are in the valence dimension. With respect to the baseline, Kuppens et al. (2010) reported significant correlations with neuroticism, extraversion, positive affect, negative affect, and satisfaction with life. The current analysis only found positive affect a meaningful covariate. While the directions of the regression weights for the aforementioned covariates were the same in the current analysis as well, their posterior credibility intervals were comparatively wide to draw any conclusions. With respect to intraindividual variability, we found only self-esteem variability as a reliable covariate, while in terms of traditional correlation measures in Kuppens et al. (2010), not only self-esteem variability, but also self-esteem, negative affect, and neuroticism were significant. Finally, Kuppens et al. (2010) did not note any significant correlations with respect to valence self-regulation, while our analysis showed that neuroticism, negative affect, and self-esteem variability all have predictive power.

These differences serve to highlight the importance of handling of parameter uncertainty across model components: While the original two-stage analysis disregarded each parameter's estimation uncertainty (by collapsing an entire posterior distribution into a single measurement point), our analysis was able to account for the posterior uncertainty in each parameter individually. As a result, outlying parameter estimates that may have driven a two-stage correlation might be down-weighted to make the correlation disappear. Alternatively, parameters central in the distribution might be down-weighted, bringing a previously unobserved correlation to the surface. The propagation of uncertainty in parameter estimates is a considerable advantage of the hierarchical Bayesian approach applied broadly.

Discussion

The HOU process model is a psychometric modeling tool that can be applied to various phenomena that are assumed to change dynamically over time. Through an example application, we demonstrated how various aspects of the temporal change mechanism can be explored, including associations between parameters of intraindividual change and exploratory variables. We note that if—as in our case—these specific relationships between model parameters and predictors were not hypothesized a priori, such results should be considered exploratory, and it is important to replicate them in a confirmatory setting before they can give rise to strong theoretical claims.

Intensive longitudinal data are expected to become more common with the increased availability of technology for collecting ecological repeated measures data. Methods for the analysis of such data are therefore of great interest to both methodologists and applied researchers. Substantial contributions of the HOU model to emotion and personality psychology involve separating substantively different mechanisms underlying observed scores. For example, variability measured through experience-sampling studies can be decomposed into measurement

error and person-specific dynamical patterns in terms of intra-individual variance and self-regulation. Moreover, the bivariate aspect of the framework allows us to take dependency between two longitudinally measured variables into account, along with studying interindividual differences in terms of synchronicity parameters.

We further demonstrated how individual difference can be explained through the addition of meaningful covariate covariates. The ability to regress model parameters onto covariates in a single step increased the accuracy of the estimated regression coefficients. We expect that these desirable properties, together with a user-friendly parameter estimation implementation, will facilitate application of the model among applied researchers.

Finally, we would like to address the question of study design. The presented model is most useful for intensive longitudinal data: data from several individuals, with more than a handful of data points each. Ideally, data have some degree of variance: The model is not ideal for measurements that only take one or two different values. If data do not contain enough information for efficient estimation of the model, convergence issues and/or large uncertainties in the parameter estimates may occur.

Article information

Conflict of Interest Disclosures: Each author signed a form for disclosure of potential conflicts of interest. No authors reported any financial or other conflicts of interest in relation to the work described.

Ethical Principles: The authors affirm having followed professional ethical guidelines in preparing this work. These guidelines include obtaining informed consent from human participants, maintaining ethical treatment and respect for the rights of human or animal participants, and ensuring the privacy of participants and their data, such as ensuring that individual participants cannot be identified in reported results or from publicly available original or archival data.

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